Detailed contents of

Crash Course

in

Foundational Mathematics

(For Diploma level courses)

The Punjab State Board of Technical Education and Industrial Training, Chandigarh

Crash Course in Foundational Mathematics

Subject- Applied Mathematics

1. Context

The Punjab State Board of Technical Education (PSBTE) is an autonomous statutory authority created under The Punjab State Board of Technical Education & Industrial Training 1992 Act for regulating and controlling academic standards in Institutes of Technical Education and for making admissions & conducting examinations in Polytechnics and Industrial Training Institutes located in the state of Punjab and Chandigarh. During the course of delivery, it was realised that the students were facing issues with their Mathematical skills and concepts. Reason for such a low performance lies in the three broader areas-

1) **Students background-**
   Most of the students come from non-mathematical Intermediate background, which require refreshing their basic concepts. Majority of the students represent socially and/or economically backward communities, and they have to work with their parents for the survival.
2) **Syllabus considerations-**
   Faculties have to complete the syllabus which comprise of grade 11^{th} and 12^{th} level Mathematics in 50-55 hours which is very less.

3) **Assessment process-**
   The assessment of the students is designed and developed from outside and hence the context and current syllabus covered end up with a gap. In some of the polytechnics the result of such assessments comes as lower as 5% only.

Under the given context, PSBTE decided to design and develop a 10 days (15 hours) Crash course for Mathematics, which will be delivered to the students immediately after their joining.

2. Objectives

- Provide opportunity to the newcomer students to refresh their concepts and skills of Mathematics in an innovative manner.
- Equip faculties with effective teaching approaches and let them reflect on their teaching methodologies.

3. Duration of the course

- Total number of periods – 20 (18 content + 2 revisions)
- Number of hours per period – 45mins
- Total number of hours for the course – 15hrs

4. **Theme wise period allotment**
   - **Number system – 2 periods**
     Period 1. Number sets
     Period 2. Operations and types
   - **Algebra – 6 Periods**
     Period 1. Need and importance
     Period 2. Understanding expressions
     Period 3. Polynomials
     Period 4. Quadratic equations
     Period 5. Solution of equations – Linear equation in 1 variable, quadratic equation
     Period 6. Square and square roots and Algebraic identities
   - **Revision – 1 Period**
   - **Coordinate Geometry – 5 Periods**
     Period 1. Need and importance
     Period 2. Ordered pairs
     Period 3. Linear equation in two variables
     Period 4. Type of lines and slope
     Period 5. Distance formula, section formula
   - **Trigonometry – 5 Periods**
     Period 1. Angles and measurement
     Period 2. Triangle and their properties
     Period 3. Trigonometric ratios
     Period 4. Standard Angle Ratios and Trigonometric Identities
     Period 5. Application of trigonometry
   - **Revision – 1 Period**

5. **Guidelines for the faculties to implement the Crash course**

- The Crash course is meant to create interest of the students in Mathematics and also to refresh their concepts and skills so that they get prepared for their respective diploma courses.
- Course comprise of 20 periods in all which has 18 periods for content delivery and 2 periods for review and support to the students.
- Faculties are expected to do the checking of assignments and based on the analysis identify the hard areas which will be discussed during the review period.
- The themes, topics and sub topics are identified considering the ground level realities and challenges.
- The Crash course needs to be interpreted as a flexible framework, allowing adaptation to institutional contexts, student needs, and individual teaching styles.
- The Idea behind developing the activities is to encourage innovation in teaching, hence faculties are expected use creative teaching methodologies, real-world applications, and interdisciplinary integration.

- For students, the design of the course fosters critical thinking and problem-solving, moving beyond procedural teaching to emphasize conceptual understanding and analytical reasoning.
- Faculties can customize content delivery to accommodate diverse learner profiles, varying levels of prior knowledge, and subject specializations within diploma programs.

Assessment:

- Each assignment will be of 5 marks, making it a total of 90. Faculty has to check the responses of each student, keep the record of the same with them and mark the score of each assignment in their registers.
- During the delivery of the course, monitoring will be done in which the record, registers and interaction with students will be done by the designated monitoring group/individual.
- The current internal assessment is of 50 marks comprising 30 marks (which is 60% of total internal assessment) for house test. With this Crash course, out of these 30 marks 10 marks will be decided from percentage received in Crash course and remaining 20 marks will be of house test. E.g. if a student received 40% marks in Crash course, s/he will get 4 marks out of 10 added in the house test. (Reference attached at flag A).
- Punjab State Board of Technical Education and Industrial Training can check the above said records at any point of time.

Theme (Number System)

Objective: Students will be able to understand

- Understand the rationale behind different number sets (up to Real number and excluding irrational numbers) and identify their properties
- Establish conceptual understanding of operations under different number sets using rules like BODMAS.
- Strengthen their skills in comparing and ordering numbers based on type and value.

Period 1-

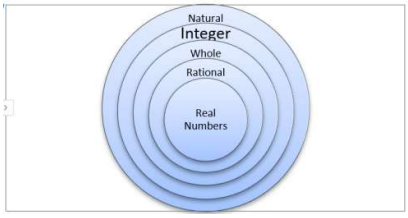
Topic- Number set

Sub topic- Natural, whole, integers, rational and, Real numbers

Process:

Activity 1:

- Faculty will draw a diagram representing the relation between the number sets incorrectly. One example is given below-



- Faculty will ask the students to analyse the diagram and share if it is correct and share what need to be corrected and why?
- Faculty will finally consolidate and correct the diagram and explain.
- Now, faculty will draw a line on the board and mark zero in between the line. Faculty will then, invite the students one by one and ask them to place the given number on the number line. E.g.
    - 5
    - 5/3
    - -5
    - -5/2
- Faculty will ask the students to compare the given numbers and their logic for the same.

Consolidation

- The faculty will guide students in summarising the hierarchy and relationships between number sets:
    - Natural ⊂ Whole ⊂ Integers ⊂ Rational ⊂ Real
- Key properties of each number set will be reinforced:
    - Natural Numbers: Used for counting (1, 2, 3…). Foundation of arithmetic.
    - Whole Numbers: Include 0, essential for concepts like place value and basic operations.

- Integers: Extend whole numbers to include negatives — important in contexts like temperature, profit/loss, elevation etc.
- Rational Numbers: Include fractions and decimals that terminate or repeat — used in measurements, ratios, and everyday calculations.
- Real Numbers: Encompass all rational and irrational numbers — form the basis of measurement, continuous data, and advanced mathematics.

Assignment:

Q.1. Watch the video and write down your understanding on the same in 100 words? The link is as given below-

https://youtu.be/ZZfQSJBTBhw?si=1L3rCXCd6smNGBRC

Period 2

Topic: Operations on numbers

Sub topic: Operations under different number sets (up to Real number and excluding irrational numbers)

Materials Required: Worksheet (as per the number of students)

Process:

Activity:

- Faculty will distribute a worksheet to the students and give them 10-15 mins time to complete it.
- The worksheet includes the given questions-
  Mark √ or X in the blank boxes in the table below

Operation/Number sets	Natural numbers	Whole Numbers	Integers	Rational Numbers	Real Numbers
A+B = B+A					
A – B = B – A					
A X B = B X A					
A ÷ B = B ÷ A					
A+(B+C) = (A+B)+C					
A-(B-C) = (A-B)-C					
AX(BXC) = (AXB)XC					
A÷(B÷C) = (A÷B)÷C					

- Faculty will now take each question one by one and ask the students to exchange their sheets with peers and check their responses.
- Faculty will share the following situation with the students and ask them to analyse and share which student has correct response according to you-
  Question: 2 + 4 ÷ (30 + 6) × 9

  Response -

Student A. 54
Student B. 83
Student C. 1.5
Student D. 3

- After collecting the response faculty will discuss each response with the students what was the mistake and how it changed the response.

Consolidation:

- Faculty will share the importance of properties of the numbers and also rules of BODAMS

Importance of Properties of Numbers

The properties of numbers are fundamental tools that help us understand, simplify, and solve mathematical problems more efficiently. Their importance lies in the following:

1. Builds a Strong Foundation:
   Understanding how numbers behave under different operations helps students make sense of arithmetic, algebra, and beyond.
2. Helps in Simplifying Calculations:
   - For example, using the commutative property ($a + b = b + a$), we can rearrange numbers to make mental math easier.
   - The distributive property is essential for expanding expressions and simplifying equations:
     e.g., $3 \times (4 + 5) = 3\times4 + 3\times5$.
3. Essential for Algebra:
   These properties are the rules that govern algebraic manipulation, factorisation, and solving equations.
4. Logical Reasoning and Proofs:
   Many mathematical proofs and problem-solving strategies are based on understanding and applying these properties correctly.
5. Avoids Misconceptions:
   Recognising that subtraction and division are not commutative or that irrational numbers behave differently helps prevent common errors.

Importance of BODMAS Rules in Mathematics

BODMAS stands for: Brackets then Orders (powers, roots) then Division then Multiplication then Addition and then Subtraction.

It defines the correct order of operations when solving mathematical expressions.

Why is BODMAS important?

1. Ensures Accuracy and Consistency: Without BODMAS, the same expression could give different answers depending on who solves it. For example:
   $6+2\times3=6+6=12$ $6 + 2 \times 3 = 6 + 6 = 12$ (Correct)
   If we don't follow BODMAS: $(6+2)\times3=8\times3=24$ $(6 + 2) \times 3 = 8 \times 3 = 24$ (Incorrect)

2. Foundation for Algebra and Equations:
   When solving expressions with variables and brackets, BODMAS provides a structured way to simplify.
3. Applies in Real-World Scenarios:
   In science, finance, and coding, the order of operations ensures precise calculations (e.g., formulas in spreadsheets or equations in physics).
4. Teaches Logical Sequencing:
   BODMAS encourages structured thinking — a core mathematical skill that extends to data handling, geometry, and logical reasoning.

In summary, properties and rules form the grammar and rules of the language of mathematics. Mastering them is essential for accurate problem-solving, higher-order thinking, and utilize it in real life too.

Assignment:

Q.1. Develop three questions based on BODMAS rule and solve them?

Theme Algebra

Objective: Students will be able to understand

- the algebraic way of thinking in solving some problems
- difference between Algebraic expressions and Polynomials
- operations of polynomials
- Linear Equation in one variable and Quadratic Equation in general form
- zeros of polynomials
- ways of solving Linear equation in one variable and Quadratic equation
- relationship between square and square roots and Factorization of Algebraic expressions

Period 1:

Topic: Need and importance of Algebra

Materials Required:

Video link: http://www.youtube.com/watch?v=NkQeTLHfDTk (simple pattern for generalization can be drawn on the board if IT facility isn't available)

Process:

Activity:

- Faculty will show the video as given in the material list and the students will be asked to generalize the pattern. (simple pattern can be drawn on the board also)
- Faculty will make the pair of students and ask them to solve the problems like:
  - 4 samosas and 5 teas cost Rs 37. But 5 samosas and 4 teas cost Rs 35. How much does a samosa cost?
  - A mother is 30 years older than her daughter. 10 years ago, the mother's age was 4 times that of her mother. How old is the mother now?
  - Keeping the perimeter of a rectangle constant, if length changes, what happens to the area?

Key discussion points/questions:

- How have they generalized the pattern?
- Try to solve the given problems with multiple ways

Consolidation:

Faculty will consolidate to arrive at the conclusion that the algebraic way is less tedious less time consuming, a powerful & useful tool to solve the problems. This brings out the importance of algebraic way of thinking. Many people think that the most useful part of mathematics is arithmetic- if you learn arithmetic, it is enough. They feel that arithmetic is useful, algebra is

bookish- algebra is only for those want to go into higher studies. Actually, arithmetic and algebra are very closely related. In general, you could say that algebra is a way of thinking about arithmetic that makes arithmetic more useful, more powerful. But first let us try and understand the typical differences between algebra and arithmetic, between the algebraic way and the arithmetic way of solving problems.

Assignment:

Q.1. Design two problems and solve using algebra and arithmetic?

Period 2

Topic: Understanding expressions

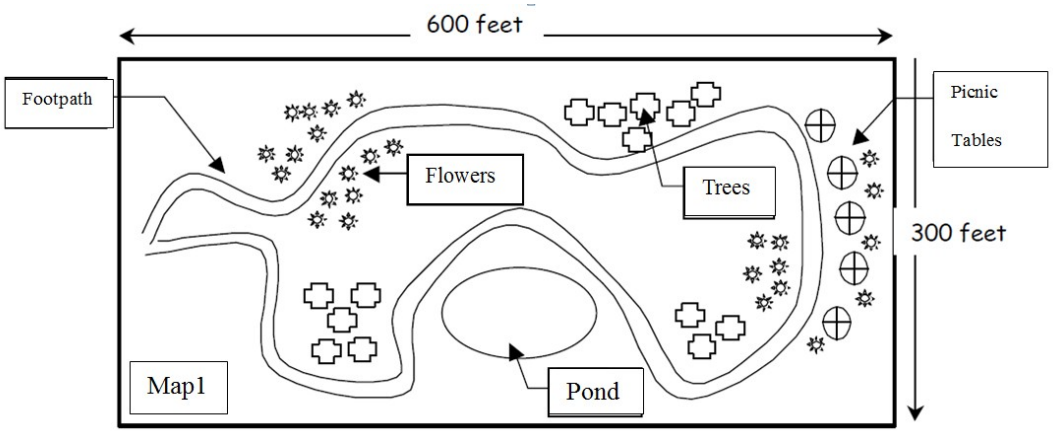
Sub topic: algebraic expression, constant, variable & unknown, term, like & unlike term, numerical co-efficient & algebraic co-efficient in a term

Materials Required: Map of a garden

Process:

Activity:

- Faculty will draw the given map on the board.
- Faculty will divide the students into groups of four.
- Each group is given a map.
- The group is asked to answer the questions in the form of "C" for the constant, "U" for the unknown & "V" for the variable by referring to the map given followed by discussion in their groups.
- Faculty will discuss it in large group by taking 1-2 responses.



Key discussion points/questions:

The garden in the picture above is open every day from 10:00 am to 05:00 pm. It costs Rs. 20 per visitor to walk through the garden. Mark C, V, U for the given cases:

- Area of the garden
- The number of people who visit the garden each day
- The amount of sunlight that falls on the garden each day
- The number of hours the garden is open each day

Consolidation:

- Faculty will explain the above cases as given below:
  - If we observe the map, the length and the width are given. The area is to be calculated using the given information. Even though the area remains constant, it is an unknown until we calculate its value by substituting the values of length and the width in the formula and multiplying them. Ex. X + 3 = 8, here the value of x will be known only after solving the equation. The value of x is 5 which is a constant but x remains as an unknown until we solve it.
  - The number of people that visit the garden each day may be any number which varies each day. Hence, it is a variable.
  - The sunlight which falls on the garden each day is a variable it depends upon factors like weather conditions which varies continuously with time. Since the context is not clear or fixed hence it is a variable.
  - The number of hours the garden is open each day is fixed and it is a constant. Even if it changes once or twice in a year by the management then also for a particular day it is fixed.
- Faculty will do the consolidation of this activity by explaining
  - A constant is a fixed quantity for example in the map given above the length of the park is 600 feet which is given.
  - The unknown is a quantity which we cannot determine unless the required data is given. For example, area of the park, which can be determined by putting the given values of length and breadth in the formula of finding the area of a rectangular shape and that is fixed but not known.
  - A variable is a quantity which goes on changing depending upon the time or the day. For example, the number of persons who visit the park goes on changing every day but its value for a given date is known.
- After consolidation of the activity faculty will explain the concepts of term, like & unlike term, factors of term (coefficients) by taking suitable examples.

Assignment:

Q.1. Design your own pattern which can be extended till the n^{th}item? Write the generalized rule for the pattern keeping any variable?

Period 3:

Topic: Polynomials

Sub topic: Polynomials, difference between expression and polynomial, monomial, binomial, trinomial, terms, degrees, operations

Materials Required:

- Chalk and Board

Process:

Activity:

- Faculty will explain the polynomials in relation to last period on algebraic expression. S/He will write few examples of equations on the board and asks the students to identify the similarities and dissimilarities in the given equations. List of equations:

$$x^4 + 2x^3 - 3x^2 + 5x + 2 = 0$$
$$x^3 + 4x^2 - 4x + 3 = 0$$
$$x^2 + 3x + 5 = 3x + 3$$
$$2x + 5 = 4x - 6$$

- Faculty will ask the students to write the point of similarity and dissimilarity in the below table given on the board-

Similarities	Dissimilarities

- Faculty will take 2-3 responses and will try take them to the meaning of degree of an equation.

Key discussion points/questions:

- Discussion on the similarities and dissimilarities in the equation
- Discussion on terms, number of terms

Consolidation:

- Faculty will conclude the discussion in the form of table given below on the board-

S.No.	Degree of the Equation	Nomenclature	General Form
1	One	Linear equation	$ax+b=0\ ;\ a\neq 0,\ a\ \&b\in R$
2	Two	Quadratic equation	$ax^2+bx+c=0\ ;\ a\neq 0\quad a,b\&c\in R$
3	Three	Cubic equation	$ax^3+bx^2+cx+d=0;\ a\neq 0\ a,b,c\&d\in R$
4	Four	Bi- quadratic equation	$ax^4+bx^3+cx^2+dx+e=0;\ a\neq 0\ a,b,c,d\&e\in R$

- S/He will also explain that a polynomial is a specific type of algebraic expression where the exponents of the variables are non-negative integers. In simpler terms, all polynomials are algebraic expressions, but not all algebraic expressions are polynomials.
- After consolidation of the activity faculty will explain other operations by taking suitable examples and reference from previous periods of number system.

Assignment:

Q.1. Write/create five real life problems in the form of polynomials?

Period 4:

Topic: Quadratic Equation

Sub topic: Importance of Quadratic Equation, standard and pure form of QE and real-life situations

Materials Required: Worksheet

Process:

Activity:

- Faculty will divide the students into groups.
- Worksheet will be given to each group and ask them to work. (Faculty can write the questions of worksheet on the board)
  - Suppose a farmer has a square field. Derive the formula for the amount of crops grown in the field in terms of side length of the field? (Assume the amount of crop which can be grown in a unit square field is 'm units'.)
  - A charity trust decided to build a prayer hall having a carpet area of 300 m^2 with its length one meter more than twice its length. What should be the length and breadth of the hall?
  - The product of three consecutive number is 494. Find the numbers.
- Faculty will take some responses after 15 minutes

Key discussion points/questions:

- How to classify these equations?
- Rationale for classification.

Consolidation:

- Faculty will take care of that the students should write only the mathematical form of the situations and not solve them on the blackboard. S/He will take one example of linear equation in one variable and then classify these equation (linear equation in one variable, quadratic, cubic and linear equation in two variable) and explain the reason.
  - Linear equation in one variable – it contains one variable and highest power/degree of the variable is one.
  - Quadratic equation – it contains one variable and highest power/degree of the variable is two.
  - Cubic equation – it contains one variable and highest power/degree of the variable is three.
  - Linear equation in two variable - it contains two variable and both with degree one.
- We have seen that many real-life problems when converted into mathematical form takes the form of quadratic equation. Some of them may be modelled using two variables but if the any one given relation contains multiplication of unknown quantity, then it can be converted into quadratic equation.
- To find the solution of such mathematical problem we need to understand quadratic equation and ways to solve quadratic equation.
- The equation of the form $ax^2 + bx + c = 0$, Here a, b and c are real arbitrary constants is called as the standard form of Quadratic equation. Here the highest power of unknown x is 2, which determines the degree of the equation. Essentially to be quadratic the value of 'a' cannot be zero (a ≠ 0). For example, $2x^2 + 3x + 8 = 0$, $x^2 + 5x = 0$, $x^2 + 8 = 0$ etc. are quadratic equations.
- From the standard equation when b = 0, the equation transforms into pure form.

$$ax^2 + c = 0$$

Assignment:

Q.1. Write at least 3 quadratic equations and 3 non-quadratic equations from real life problems?

Period 5:

Topic: Solution of equations

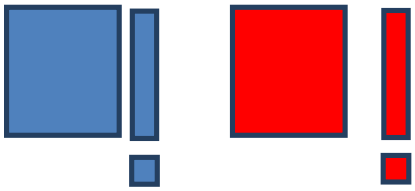
Sub topic: Solution of Linear equation in one variable, Discriminant, Nature of roots, solution of QE by factorization, Quadratic formula and Graphical method, Meaning of roots of QE and need of complex numbers

Materials Required: Tiles (can be drawn on the board)

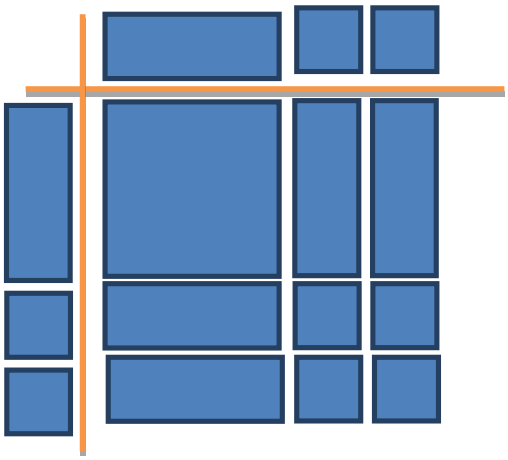
Process:

Activity:

- Faculty will draw the symbol for x^2, x and ones on the board as given below where blue indicates for positive and red for negative.



- Faculty will take an example of x^2 + 4x+ 4and arrange them to form a square/rectangle as given in the picture.



- Faculty will explain that the side is x + 2 on each side will be the factor of the equation, which will give the solution of this equation as -2.
- Similarly, s/he will take an example of x^2 -4x+ 4and arrange them to form a square/rectangle to explain.
- Students will be given some problems to solve by drawing like $x^2 - 3x + 2 = 0$
- Now Faculty will explain the QE formula to the students as given below:
  - The standard form of Quadratic equation$ax^2 + bx + c = 0$.
  - From the standard form of Quadratic equation subtract c on both sides.

$$ax^2 + bx = -c$$

  - Multiply both sides by 4a

$$4a^2x^2 + 4abx = -4ac$$

  - Add b^2 to complete the square

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

  - Rearrange the equation

$$(2ax + b)^2 = b^2 - 4ac$$

- Taking square root

$$(2ax + b) = \pm\sqrt{b^2 - 4ac}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Hence the quadratic equation $ax^2 + bx + c = 0$ has two solutions $x_1 = \frac{-b + \sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2-4ac}}{2a}$.

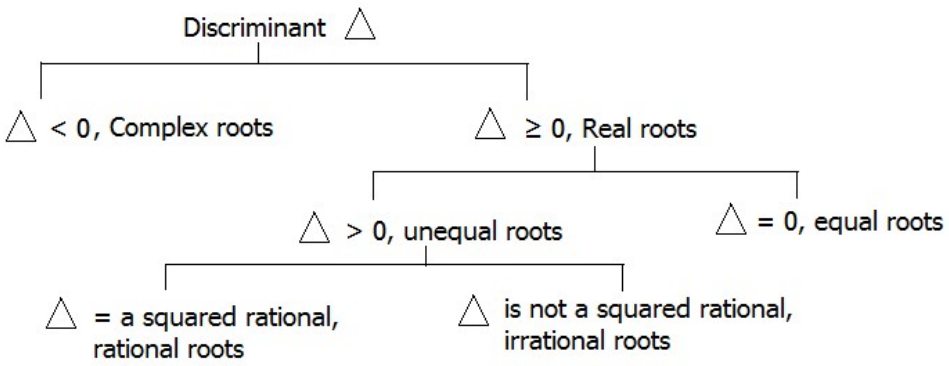
- Similarly, faculty will explain the other topics related to QE.

Key discussion points/questions:

- How to arrange/draw the square/rectangle to get the factors?
- What challenges they have faced?

Consolidation:

- Faculty will explain that the solutions of a quadratic equation which satisfies the equation are called as roots. As discussed in the previous section the general formula used to determine the roots of a quadratic equation $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$. The expression under the square root in the formula $(b^2 - 4ac)$ is called the discriminant. It is denoted using Δ (delta) which is a Greek symbol or using the capital letter D. So $\Delta = b^2 - 4ac$. The discriminant is a very useful concept. It helps to identify the nature of solutions of a quadratic equation. Discriminant may be more than, equal to or less than zero. Here a, b, c are real numbers.



Assignment:

Q.1. Write real life problems related to Quadratic Equations and solve them either by Tiles method or by Quadratic formula?

Period 6:

Topic: Square, square roots and Algebraic Identities

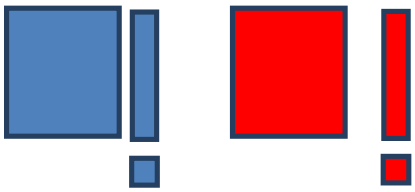
Sub topic:square, square roots, algebraic Identities like $(a+b)^2$, $(a-b)^2$, a^2-b^2, etc

Materials Required: Tiles (can be drawn on the board)

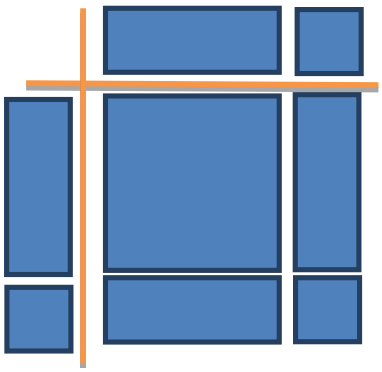
Process:

Activity:

- Faculty will draw the symbol for x^2, x and ones on the board as given below where blue indicates for positive and red for negative as done in last period.



- Faculty will take an example of $(x+1)^2$and do the multiplication with the help of tiles which will be drawn as



- Faculty will take other example like $(x+2)^2$, $(x+3)^2$and do the multiplication with the help of tiles.
- Faculty will write them in a table and students will be asked to find the pattern and to generalize it in the form of identity.
- Now students will be do it similarly for $(a-b)^2$ by drawing.
- Faculty will explain the other identities and square, square roots in relation to the above discussion.

Key discussion points/questions:

- What pattern has been identified and how?
- What challenges they have faced?

Consolidation:

- Faculty will explain all identities by multiplication and factorization like for (a-b) (a-b), factorization of a^2-2ab+b^2. Faculty will also discuss on their use in real life for calculation and other concepts.

Assignment:

Q.1. Write five real life situations, solve them using direct method and using identities for calculation?

Theme: Coordinate Geometry

Objective: Students will be able to understand

- The need and importance of Coordinate Geometry
- The concept of linear equation in two variables
- The meaning of ordered pair and their plotting in Coordinate Geometry
- The nature of solutions of Linear equation in two variables and its solution
- The concept of slope and the various equations of lines including parallel and perpendicular lines
- The basic formula like slope formula and distance formula

Period 1-

Topic- Need and importance of Coordinate Geometry

Sub topic- Reference frame of Coordinate Geometry (Quadrants), location of any point, abscissa and ordinate

Process:

Activity:

- Faculty will ask the students to share their understanding of sharing address of any particular thing to a stranger.
- S/he will take an example of any famous place nearby and ask the students to write the address so that anyone can reach their easily.
- Faculty will take the responses from 4-5 students and try to figure out the differences in their communication.
- Now, one student will go out of the classroom and the remaining students will write the instructions to get the location of an object in the class so that the student not present in the class will be able to locate that particular object.
- Faculty will take the instructions/location identification points from a few students and will ask that particular student to come in the class and to identify the object's location on the basis of instruction.

Key discussion points/questions:

- How to share the location?
- What key points/reference will be taken?
- How to make the reference so that everyone is able to understand with uniformity?

Consolidation:

Faculty will try to establish the need and importance of Coordinate Geometry with precise ordered pair for any object. Co-ordinate geometry is essential for describing geometric shapes and their properties using algebraic equations. It provides a way to represent geometric figures on a plane or in space using coordinate, enabling us to analyse and solve geometric problems using algebraic methods. The bridge between geometry and algebra is crucial in various fields. Applications in various fields: -

- Co-ordinate geometry is used in physics to describe the motion of objects, analyse forces and model physical phenomena.
- Everyday life: from maps on our phones to understanding directions, co-ordinate geometry plays a role in a daily life.
- Engineering: Co-ordinate geometry is used in various engineering fields such as Architecture, Mechanical Engineering and Electrical Engineering for design, analysis and modelling.

S/he will also try to link the Latitude and Longitude reference to get the coordinates of any country/place. Examples of coordinates used by army to locate any particular target to attack. Faculty will now explain the coordinate geometry reference with all four-quadrant including abscissa and ordinate, equation of x-axis and y-axis, etc.

Assignment:

Q.1. Describe the need of co-ordinate geometry in any one of the following fields:

- Sports
- Construction
- Transportation

How the coordinate geometry helps in these fields and what will happen if we don't have any knowledge of Coordinate geometry?

Period 2-

Topic- Ordered Pair

Sub topic- understanding of ordered pair, abscissa and ordinate of any ordered pair, plotting of ordered pair

Materials Required: Two dice, a coordinate plane (a graph paper or a pre mode game board or a grid on notebook), marker

Process:

Activity:

- Faculty will divide the students into the group of four.
- Each group will be given a coordinate plane and two dice of different colour/size.
- One colour of dice will represent x axis whereas another colour will represent y axis.
- Each group consisting of four students will get one chance to roll and to get an ordered pair (x, y).
- Each student will plot their ordered pair in their group.
- Now faculty will ask 2-3 groups to come and draw their coordinate plane on the board.
- Faculty will take ordered pair of other quadrants like (-2,3), (-2, -2), (2.-3) and ask the groups to plot same on their coordinate plane.
- Ask 2-3 groups to present it on the board and all other groups will get it corrected/feedback.

Key discussion points/questions:

- How to plot a point?
- Which will be plotted first, x or y?

Consolidation:

Faculty will try to explain the ordered pair, abscissa and ordinate of any ordered pair, plotting of ordered pair in different quadrants. Focus should be given in order to have understanding of plotting in all four quadrants.

Assignment:

Q.1. Draw three figures i.e. square, rectangle and triangle on a graph paper and write their ordered pair?

Period 3

Topic- Linear Equations in two variables

Sub topic- Linear Equation in two variables, its plotting, solution by graphical method, types of solution of Linear Equation.

Materials Required: Graph paper

Process:

Activity:

- Faculty will take two cases from real life and will ask the students to discuss it in pairs.

Case 1:

Sameer went to market to buy potatoes and tomatoes. Rate of potatoes is Rs. 30 per Kg and tomatoes is Rs.40 per kg. How much potatoes and tomatoes did he buy for Rs.240?

Case 2:

In a cricket match HarmanpreetKaur scores 60 runs through fours and sixes. What is the total number of fours and sixes she hit in the match?

- Faculty will take responses from a few pair of students to know their way of thinking and the ordered pairs they have got.
- Faculty will try to jot the points/ordered pairs on the board.
- Students will be asked to do plot these ordered pairs on their graph and to get the lines.
- Similarly, students will be asked to take any equation, get their ordered pairs by putting values of x to get the values of y and plot the Linear Equation.

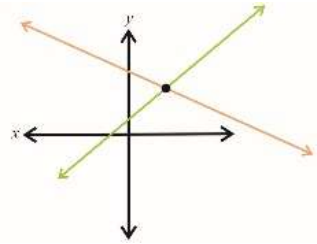
Key discussion points/questions:

- In how many ways we can get potatoes and tomatoes?
- Ordered pairs for both cases, etc
- How to get the exact solution/case of Linear Equation?

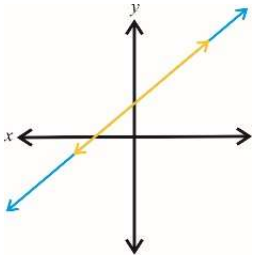
Consolidation:

Faculty will try to explain the Linear Equation in two variables, need of pair of Linear Equation in two variables to get the solution and to get the nature of solution. Faculty will explain the following for consolidation:

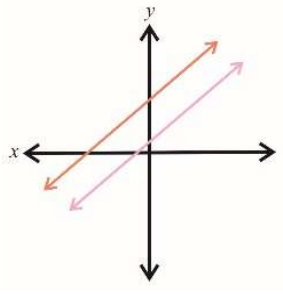
- To arrive at the particular solution at least one more equation is necessary.
- There are infinite number of solutions for a linear in equation in two variables but all the solutions are not suit for particular real-life situations.
- A linear equation in one variable has only one solution, nth degree equation in one variable has at most n solutions.
- When two linear equations plotted on a plane there are three possibilities - they intersect, they coincide or they are parallel to each other as shown below.
    - When the lines intersect it means that there is a unique ordered pair that satisfies both the pair of linear equations.
    - When the lines coincide it means that there are infinitely may ordered pairs that satisfy the pair of linear equations.
    - When the lines are parallel to each other it means that there are no ordered pairs that satisfies the pair of linear equations.
- The ordered pair/s that satisfy the pair of linear equations is/are referred to as the solution/s of the pair of linear equations in two variables.



Graph 3 (a): Intersecting lines



Graph 3 (b): Coinciding lines



Graph 3 (c): Parallel lines

Assignment:

Q.1. Frame three pair of Linear equation in two variables, plot them on the graph to get the solutions?

Period 4-

Topic- Types of lines, slope

Sub topic- Different equations of straight lines, slope, parallel lines and perpendicular lines

Materials Required: Ball and notebook

Process:

Activity:

- Faculty will take real life examples to discuss about the slope and its importance as follows:
  - What will happen if there will be no slope in washroom?
  - What will happen if there will be no slope on roof?
  - Why we have ramps in our houses?
  - What will happen if ramps are quite steep? etc
- After having these discussions faculty will take a notebook and ball to demonstrate the different inclinations of slope with the help of students. S/he will ask the students to write down the movement of ball as per different slope?
- Faculty will explain the concept of slope and steepness on the basis of above examples.

Key discussion points/questions:

- What is slope?
- What's the effect of increasing and decreasing the angle of slope?

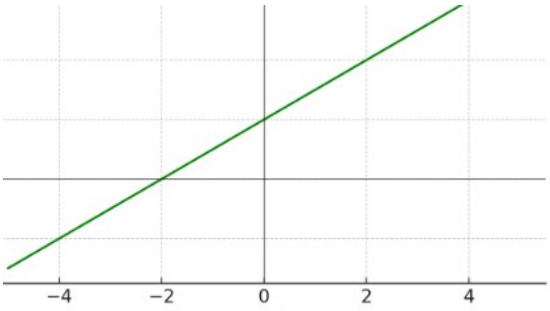
Consolidation:

Faculty will link the above activity of slope with the slope of line by showing different cases as given below:

Slope: The slope of a line is a ratio of the change in y to the change in x between any two points on the line.

Slope= Change in y/change in x

Positive slope: for a line y = x + 2



Similarly other cases will be explained. Faculty will explain about the slope formula and its linkages with tangent ratio. With the help of slope formula, s/he will explain the perpendicular and horizontal lines. At the end s/he will tell them about the different equations of lines in linkages to previous periods.

- Straight line is the shortest distance between any two given points.
- Lines that run parallel to the horizon are known as Horizontal lines.
- Lines that run perpendicular to the horizon are known as Vertical lines.
- Lines that intersect at any angle are known as intersecting lines.
- Lines that intersect at 90^0 are known as perpendicular lines.
- Lines that run in same direction and maintain a constant distance from each other are known as parallel lines.

Assignment:

Q.1. Find/draw different types of equations of lines?

Period 5-

Topic- Distance formula, section formula

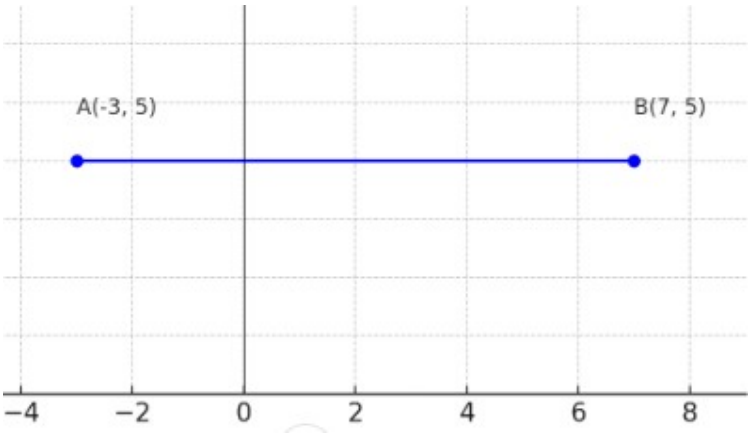
Sub topic- distance formula, section formula, midpoint formula

Materials Required: Cardboard, chart Papers, graph paper, glue, geometry box and Pen/ Pencil

Process:

Activity:

- Faculty will ask the students to paste a chart paper on a cardboard of a convenient size.
- Students will paste the graph paper on the chart paper and draw the x-axis and y-axis on the graph paper.
- Faculty will ask the students to take two points A (-3, 5) and B (7, 5) on the graph paper and join them to get a line as given in the picture.



- Faculty will ask the students to take two points A (-3, 5) and B (7, 5) on the graph paper and join them to get a line as given in the picture.
- Faculty will explain the distance formula as given below:

$$x_1 = 4 \;\; y_1 = 2$$

$$x_2 = -1 \; y_2 = 1$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(-1 - 4)^2 + (1 - 2)^2}$$

$$= \sqrt{(-5)^2 + (-1)^2}$$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

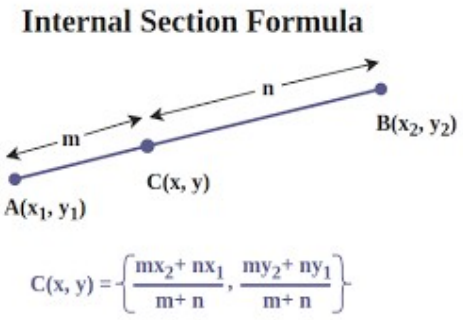
$$= 5.1$$

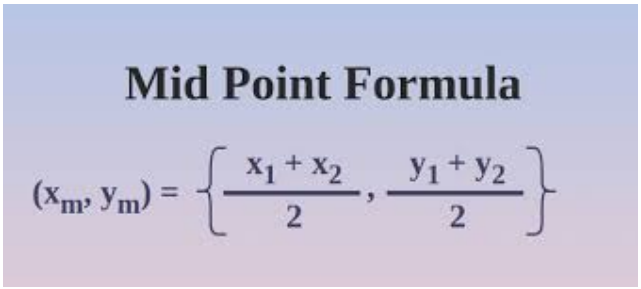
Key discussion points/questions:

- Measure the distance between two points A & B using a ruler.
- Calculate the distance AB using distance formula.
- Is the distance calculated by distance formula and distance measured by the ruler same?

Consolidation:

Faculty will try to explain the generalized form of distance formula by taking some other examples. Similarly, s/he will try to explain section formula and mid-point formula as given below:





Assignment:

Q.1. Plot at least three or more points on the graph and calculate the distance using ruler and the formula? Verify?

Theme: Trigonometry
Objective: Students will be able to-

- Identify, classify and measure angles in their surroundings using a protractor.
- Understand and tell different types of triangles and their basic Properties.
- Understand trigonometric ratios (sine, cosine, tangent) using right-angled triangles, derive their values for standard angles, understand trigonometric identities.
- Enhance students' abilities to use trigonometry as a tool in solving simple problems and situations.

Period 1
Topic: Angle and its Measurement
Sub-topics:

- Types of angles: acute, right, obtuse, straight, complete.
- Measuring angles using a protractor.
- Estimating Angle size by observation.

Materials required: Protractors, Pencil, ruler, Notebook

Process:

Activity 1: Warming up

- Faculty will quickly revise types of angles using drawing on board.
  - Acute ($<90^0$)
  - Right ($=90^0$)
  - Obtuse ($> 90^0$ and $< 180^0$)
  - Straight ($=180^0$)
  - Complete angle ($=360^0$)

(Note: Here the faculty can also introduce to the another system of measurement like $180^0 = \pi$, $90^0 = \pi/2$, $45^0 = \pi/4$, $30^0 = \pi/6$ etc.)

Activity 2: Angle Hunt

- Faculty will let the student explore the classroom in pair or small groups and ask for following task-
  - Ask them to find at least 5 different types of angles around them and their usage. (Examples: Corners of books, stairs, doors etc.)
  - Students will then measure the angles using protractor and finally draw or sketch the angles.
  - Students will record each angle's location, type and measurement in their copies.

Activity 3: Group sharing & Discussion

- Faculty will invite each group to shares 1 or 2 interesting places where they found the angles and discuss following key questions

Key Questions:

- Which angle was smallest or longest?
- Any interesting facts they want to share?
- Any mistake realized while measuring the angles?

Consolidation:
It will be done along with period 2.

Assignment:

Q.1. Draw the given angles with freehand and then verify the same using protractor-

- 30^0
- 45^0
- 120^0
- 135^0
- 150^0
- 240^0
- 330^0

Did you observed anything or find any relation between pair of angles like 30^0 and 330^0 are measuring 30units but in opposite directions? Can we write 330^0 as -30^0?

Period 2

Topic: Triangles and their Properties

Materialsrequired:

Colored Papers (or cut-outs of different types of triangles)

Process:

Activity 1: Types and Properties of Triangles.

- Faculty will divide students in small groups and ask them to cut 4 different types of triangles.
- This will be followed by students measuring the angles and sides of each triangle.

Triangle	Side 1	Side 2	Side 3	Angle 1	Angle 2	Angle 3	Type of Triangle
Triangle 1							
Triangle 2							
Triangle 3							
Triangle 4							

Activity 2:Pythagoras Theorem

- Faculty will provide hands on demonstration to the students through providing right angled triangle (Cut-out) with given side lengths.
- Faculty will ask the students to verify the same through measuring the sides and check if the square of the sides have any relation.
- After the discussion with the students, faculty will articulate the Pythagoras theorem, “in a right-angled triangle, square of hypotenuse is equal to the sum of the squares of the other true sides.”

Consolidation:

Types and Properties of Triangles

A **triangle** is a polygon with three sides, three angles, and three vertices. The sum of the interior angles of any triangle is always 180°.

Types of Triangles

1. Based on Side Lengths

Type	Description	Example Diagram
Equilateral	All three sides are equal; all angles = 60°	△ABC with AB = BC = CA
Isosceles	Two sides are equal; base angles are equal	△PQR with PQ = PR
Scalene	All three sides are of different lengths	△XYZ with XY ≠ YZ ≠ ZX

2. Based on Angles

Type	Description	Angle Range
Acute Triangle	All angles are less than 90°	∠A, ∠B, ∠C < 90°
Right Triangle	One angle is exactly 90°	∠A = 90°
Obtuse Triangle	One angle is more than 90°	∠A > 90°

Properties of Triangles

- **AngleSum Property**:
  The sum of the interior angles is always **180°**.
- **Pythagoras Theorem**:
  $a^2+b^2=c^2$
  where a and b are base and perpendicular of the right triangle and c is the hypotenuse.

Assignment:

Q.1. a) Do you think (3, 4,5) is a Pythagorean triplet? Why? Share two more such triplets if the right triangle is

- Scalene
- Isoceles

b) Is it possible to have an equilateral triangle to be a right triangle? Why?

Period 3

Topic: Trigonometric Ratios

Sub-topic: Trigonometric Ratios through Pythagorean Triplets

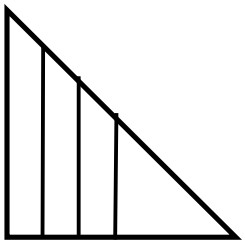
Materials Required:

Chart paper or whiteboard, geometry box having all tools

Process:

Activity:

- Faculty will demonstrate on board and ask the students to draw right triangles having 45^0, 45^0 and 90^0 angles and mark perpendiculars as P1, P2, P3 and P4.



- Faculty will ask the students how many right triangles they can see in the picture drawn. The response will be 4 (if not faculty will facilitate).
- Faculty will ask the students to measure the length of perpendicular and base of the four triangles and share their observations.
- Faculty will introduce the constant relation as Tan θ. Similarly, faculty will share the other trigonometric ratios to the students.

Consolidation:

- Trigonometric ratios are ratios of the sides of a right-angled triangle with respect to one of its acute angles. These ratios are fundamental in trigonometry and are used to relate angles to side lengths.
- For any right triangle, if we draw 'n' numbers of perpendiculars parallel to the existing perpendicular on the base intersecting the hypotenuse forming 'n' number of right triangles, the trigonometric ratios remain constant.
- There are six primary trigonometric ratios:
  - Sine or Sinθ = Perpendicular/Hypotenuse
  - Cosine or Cosθ = Base/Hypotenuse
  - Tangent or Tanθ = Perpendicular/Base (here the faculty will explain how it is related with the identity of Tan θ as Sin θ/Cos θ)
  - Cosecant or Cosec is reciprocal of sine iesinθ = 1/cscθ
  - Secant or Sec is reciprocal of cosine iecosθ = 1/secθ
  - Cotangent or Cot is reciprocal of tangent ietanθ = 1/cotθ

Assessment: It will be done along with period 4

Period 4

Topic: Standard Angle Ratios and Trigonometric Identities

Sub-topic: Deriving Trigonometric Ratios for Standard Angles (0°, 30°, 45°, 60°, 90°)

Materials Required: Graph sheets, Geometry box, pre-drawn triangle templates with marked angles, Table for trigonometric values

Process:

Activity:

- Faculty draws two right-angled triangles on the board:
- Triangle T1: angles 30-60-90 with P = 1 cm, H = 2 cm → find B
- Triangle T2: angles 45-45-90 with P = 1 cm, B = 1 cm → find H
- Students compute missing sides using the Pythagorean theorem.
- Using these, fill out a trigonometric ratio table for:

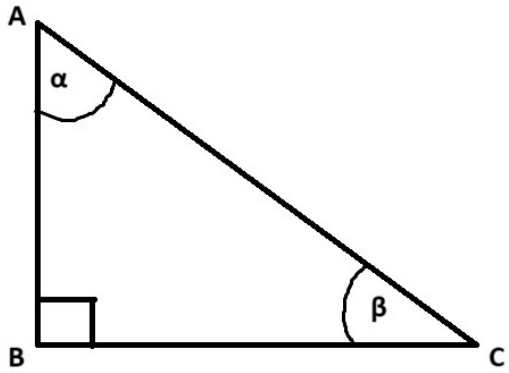
T-Ratios	**0°**	**30°**	**45°**	**60°**	**90°**
sin θ					
cos θ					
tan θ					

- Now the faculty will explain the method of finding the trigonometric identities (Pythagorean) and will also solve some question on the board so that students become familiar with the method of implementation of these identities.
  $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

Assignment:

Q.1. Find all trigonometric ratios of a right-angled triangle having both acute angles 45^0 and length of the base 7units?

Q.2. Given a right triangle ABC right angled at B, and acute angles be α and β with sides 5cm,12cm, and 13cm, find Sin α, Tan β, Cos β and Cosec β



Period 5

Topic: Application of Trigonometry

Sub topic: Angle of Elevation, angle of depression, application in real life

Process:

Activity 1:

- Faculty will, divide the students in different groups, take the students outside the classroom and ask them to measure one student's height and the length of his/her shadow in their respective groups.
- Students will record the observation as
  - Height of the student……cm.
  - Length of the shadow……. cm.
- Students will also draw a rough sketch of a right-angled triangle to model the situation, calculate the unknown side of the triangle and record all the measurements.

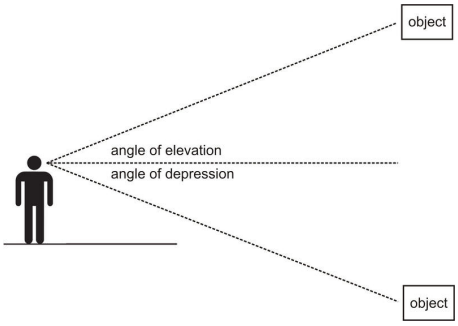
Activity 2:

Angle of Elevation:

- Faculty while keeping the same groups, willgive an example for angle of elevation and depression and its representation in diagram form.
- Faculty will then, keeping angle of elevation into consideration, assign each group an object in the school environment to measure (height of a flagpole or a tall tree or building etc.).
- Faculty will only support if felt like otherwise s/he can give small hints in the larger group.
- Students are required to measure height of the given objects by drawing the diagram representing the situation and using the t-ratios and identities.
- Finally, using trigonometric ratios (tangent, sine, or cosine depending on the known and unknown sides), they can calculate the height of the object.
- Similarly for angle of depression faculty will ask the students to choose a non-risky elevated position and measure the angle of depression to a point on the ground.
- The students will also represent the situation in diagram, label the knowns, and calculate the height of the object or the distance to the object on the ground, depending on what they are trying to find.

Consolidation:

The angle of elevation is the angle formed when an observer looks upwards at an object, while the angle of depression is the angle formed when an observer looks downwards.



Assignment:

Q.1. Observe and design two problems from your daily life context and solve the same using trigonometry?